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Design and Selection of Risk-Based Control Charts

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Abstract

The widely used classical control charts investigate manufacturing and service processes and signal their non-conformance with reliability centered approach. The application of each type of these charts has constraints that should be considered in order to get the proper result in the quality level. The violation of the constraints increase the probability of the decision errors and decrease the quality level, hence the profit associated with the decision. In this paper the authors propose a method that guides the decision maker in the selection of the control chart that fits the best to the investigated manufacturing or service process. The proposed method also modifies the selected control chart to take the uncertainty of measurement and estimation, and consequences of the decision into account. In the use of these new risk-based control charts the sample size, the frequency of sampling, and the alteration of the control limits are determined in order to maximize the expected profit of the decision.

Keywords: statistical process control (SPC), control charts, measurement uncertainty, risk.

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1. Introduction

The control charts are widely used tools of statistical process control (SPC), but their applicability is limited if the observed process not follows a Gaussian distribution. The classical SPC (e.g. x-bar, s, R) charts over- or underdo the control of the process in case of non-normality. An additional problem is that most of these charts are neglecting the financial/economic aspect of the process control.

The aim of our research is the revision of process control techniques and to develop a new method for the selection, design and fitting the control charts to the processes. Taking the measurement uncertainty and decision risk into account to select and fit the appropriate control chart to the observed process is the novelty of the proposed method.

2. Background

The applicability of control charts and tailoring them to special cases was in the focus of numerous studies recently. In Table 1 these charts are collected and categorized according to which properties of them was the emphasis on during the design course. There are solutions for the non-normality of the process or using variable control parameters but most of them work only on reliability base and not deal with the decision cost and consequences.

Table 1. Research scope of the practical implemented control charts

	Control charts			
	Reliability-centred		Risk-based	
	Constant sample size and sampling intervals	Variable sample size and sampling intervals	Constant sample size and sampling intervals	Variable sample size and sampling intervals
Normality is assumed	x-bar, s, R, CUSUM, EWMA, MA	CUSUM, x-bar, s EWMA, MA, T^2	x-bar	-
Normality is not assumed	x-bar, R, CUSUM, EWMA, MA	X-bar, CUSUM, EWMA, MA	-	-

Source: [1-15]

The purposes and goals in the chart selection worth a thorough analysis. The choice between the variable and attribute chart is part of the selection course [16]. In most of these decision models the measurability and the sample size are the main factors.

In the next section our control chart selection method is introduced that combines the previous methods and augment them with a risk-based point of view and the handling of measurement uncertainty.

3. The proposed method for control chart selection and design

The first step in the statistical process control of a production process is determination of the applicable methods. The proposed method is different from the classical one it also takes the probability distribution of the observed process, the properties of the inspection and the measurement, the parameters of the control chart and the economic consequences of the decisions into account.

Our method consists of 7 steps that are described in the following subsections.

3.1. Collecting the properties of the technological process and the investigation

The first step of the chart selection process is the analysis of the production process and definition of the inspection methods. The typical questions: What are the main characteristics of the product or the process that determine the quality (conformity) of the product? Are these characteristics directly measureable or observable? What kind of probability distribution describes well the variation of these characteristics? Is the inspection destructive to the product? This step provides vital information to the following ones.

3.2. Choosing between variable and attribute control charts

After sufficient information have been gained from the analysis of the production process the choice between the variable or control charts comes. If the technology can be described by measurement results, and not just the conformity of the process/product can be stated but the particular data is available about the deviation from the target value then use of variable charts is expedient. If the difficulties are thrown in the measurement's way attribute charts are reasonable.

3.3. Specification of the inputs and constraints for each chart

Observability of the attributes measurability of variables and the chart usability constraints are in the focal point of the next examination. The determination of the probability distribution of the variables also has a great importance. The probability distribution of the observed characteristic is the main split point if it do not follow the Gaussian distribution only the moving average and exponential moving average charts can be used from the common variable charts without transformation of the variables or parameters. The normality from the central limit theorem also depends on the sample size.

3.4. Selecting the adequate chart(s) according to the information obtained from historical data

When the charts have been investigated from the appropriate inputs view-point and the possible alternatives are selected the test of these charts on historical data is next step. With this validation the set of the charts can be narrowed down and their goodness of fit can be compared.

3.5. Selecting the constant and variable parameters

In the fifth step it can be decided whether the parameters of the chart will be constants or variables. With the variable parameters the sensitivity of the charts can be altered and the costs and reliability balanced. The sampling interval (h), the sample size (n) and the control limits (LCL, UCL) also can be variables. There are hierarchy levels according to the number of the variable parameters: all parameter is constant, all parameter is fixed (FP), one of the parameters is variable (interval - VSI, limits - VSL, sample size - VSS), two from the parameters are variable (VSSI, VSSL, VSIL) and all of the parameters are variable (VP).

3.6. Determination of cost, revenue and profit associated with the decision

In the fitting of the selected chart we take also the risk arising from the measurement and estimation uncertainty into account. To calculate the risk the costs and revenues belonging to the different decision outcomes needed to be specified. In the simplest case there are 4 outcomes: two correct decisions, the acceptance of the conforming process and the revision of non-conforming one, and there two decision errors, in type I the superfluous control is performed on a conforming process, in type II error the non-conforming process stays uncontrolled. To these outcomes profits or losses (from the difference of the revenues and costs) and the probabilities of occurrences can be assigned.

3.7. Modification of control limits taking measurement uncertainty into account

Every measurement is subject to uncertainty and the measured value y is not equal to the actual value x but theoretically y can be described as the sum of the actual value and the measurement error: $y(t) = x(t)+m(t)$ where $t \in \mathbf{N}$ is the time of the measurement. In the practice neither the actual value nor the measurement error can be known.

In the seventh step the point of the measured value is substituted with an interval and this interval is compared to the control limits – $[y(t)-K_L]\geq$LCL and $[y(t)+K_U]\leq$UCL, K_L , $K_U \in \mathbf{R}$ –in this way the risk of decision can be modified [17] by the alteration of the probabilities of the decision outcomes. The optimal values of K_L and K_U correction factors depend on the distribution of the observed process, the measurement uncertainty and the consequences of the decision outcomes and maximize the expected profit associated with the decision. These optimal values can be determined by numerical analysis or simulations. They set the balance between the decision errors (type I and type II) and minimize the total loss of these errors.

The last steps of the selection and design are tested in a practical example that is presented in the next section.

4. Practical example

The observed process is the filling of steel cartridges (cylinders) with 8 gram gas. These cartridges are used in needleless injection systems and the maximal permissible deviation from the target mass is 0.4 g.

Figure 1 shows an example of using standard moving average (MA) chart and the optimal modification with the correction components (K_{LCL} and K_{UCL}) delineated with red.

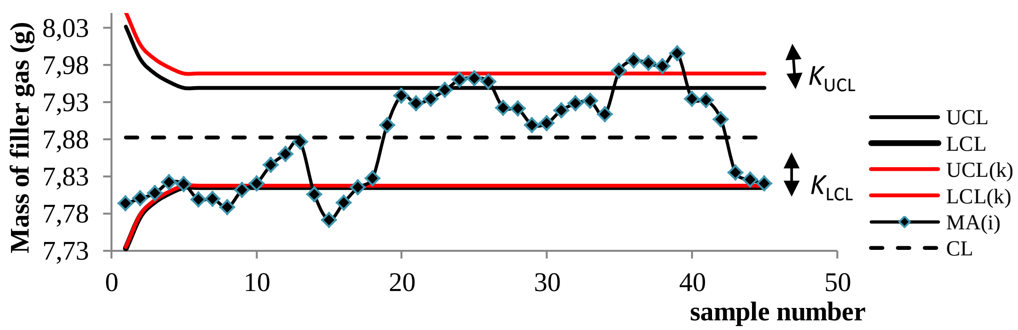

Figure 1. The moving average chart designed to the analysed process

The upper control limit of the MA chart is loosened to reduce the false positive signals and superfluous control of the process. However in the use of exponential weighted moving average (EWMA) charts (Figure 2) the upper limit needs to be stricter to avoid the occurrence of decision error type II.

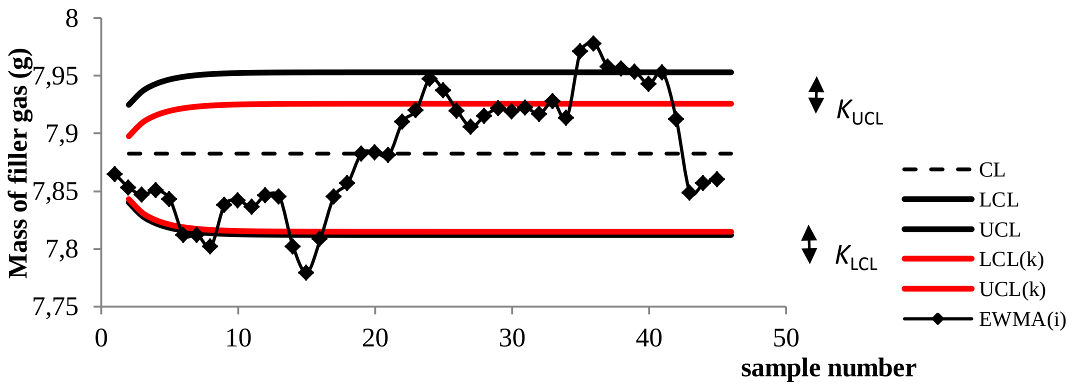

Figure 2. The exponential weighted moving average chart designed to the analysed process

The effectiveness of our method is tested with a Matlab simulation. Figure 3 shows the value of the profit as a function of correction components using MA chart for a Weibull distributed process. The profit is constant on this figure if the uncertainty is neglected, which is illustrated with the vertical red plain. The other profit shape depends on the K_L and K_U values and shows a better solution than the standard MA chart when it is above the vertical plain. The highest point of this shape gives the maximal profit and the optimal value of the correction component pair.

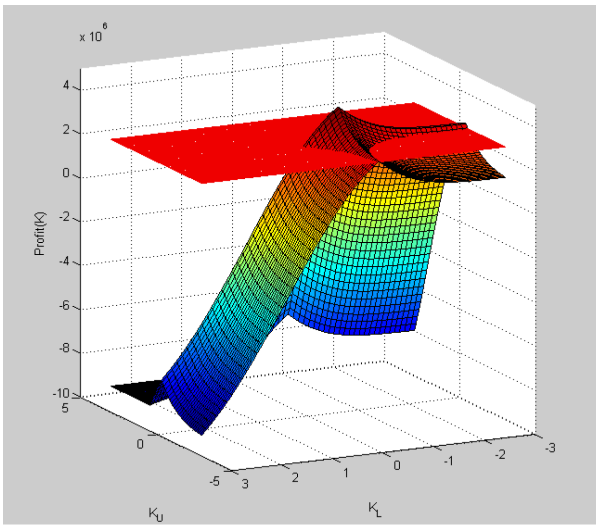

Figure 3. The trend of the profit for all samples, depending on kLSL and kUSL parameters for the moving average chart. The values of the analyzed parameter follows Weibull distribution

5. Summary

In this paper the difficulties of the control chart application were presented. Most of the control charts neglect the consequences of the decision error and assume the normality of the observed process, which increases the decision risk. The process of the selection and design of the charts is enhanced in this paper. Beyond the analysis of the observed process, choosing the controlled characteristic and define the variable and constant chart parameters the consideration of measurement uncertainty and decision risk is also part of the chart designing course.

The implementation of these steps is demonstrated on the statistical control of a gas-cylinder filling procedure. Moving average and exponential weighted moving average charts are modified in order to reduce the total cost from the decision errors. With both control chart modification the growth in the profit associated with the decision is near the 10%. Besides the consideration of uncertainty and risk the error from non-normality can be eliminated by the alteration of control limits.

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